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MIMO generalized predictive control for a hydroelectric power station

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***Abstract*--In this paper, the approach of Generalized Predictive Control (GPC) is applied to a multivariable model of the Dinorwig pumped - storage hydroelectric power station. The response of the system with constrained predictive control is compared with the classic PI controller as currently implemented. The results show that GPC offers significantly better performance across the plant's operating range. It is shown that fixed-parameter, constrained GPC produces a faster primary response when the station is operating with a single unit while preserving stability as the operating conditions change when multiple units are on-line.**

***Index Terms*-- Hydroelectric power; power station control; control applications; multivariable control; simulation; model predictive control.**

I. Introduction

THE First Hydro Company operates a large pumped storage hydroelectric power station located in Dinorwig, in North Wales. Six 300 MW rated Francis turbines, driving synchronous generators, feed power into the national grid to provide rapid frequency control. The station is supplied by a single tunnel drawing water from an upper reservoir into a manifold, which splits the flow into six penstocks, each feeding a turbine/generator to produce electrical power. Figure 1 is a schematic of the Dinorwig station; a more detailed description of the plant and its operation is given by Mansoor *et al* [1].

The generated power is varied by controlling the flow of water using a guide vane at each turbine. Every turbine drives a synchronous generator and delivery of power is governed by individual PI feedback loops on each Unit, which vary the guide vane angle and hence the flow of water into the turbine. Typically, several Units will be operated in power-control mode (sometimes referred to as 'dead-band' mode), regulating their output power to a fixed reference value in the range 150 – 280MW. When used as spinning reserve, these Units are capable of going from zero to full power output (and vice versa) in 10 - 15 seconds. At the same time, one or two Units will be operated in frequency-control mode (sometimes referred to as Part Load Response (PLR) mode). In this mode, a signal proportional to the grid frequency deviation from its 50 Hz set point is added to the fixed power reference, thus forming an outer frequency control loop.

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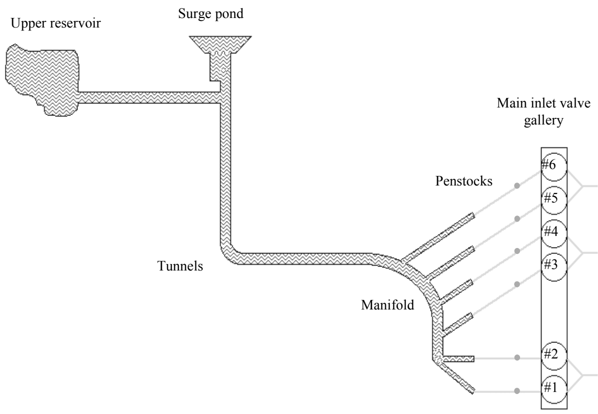


Figure 1 Schematic diagram of the tunnel and penstocks at Dinorwig (not to scale).

A. Characteristics of hydroelectric plant.

From the point of view of controlling real (as distinct from reactive) electrical power and frequency, the main features of hydroelectric plant are their non-minimum-phase (NMP) dynamics, poorly damped poles (associated with water-hammer in the supply tunnel and electrical synchronisation) and nonlinear relationships between guide vane angle, volume flow of water and mechanical power [2]. Neglecting losses, the fundamental nonlinearities are:

a. The flow (q) through the turbine depends on the guide vane angle (G) and the pressure, or head (h), according to $q = G\sqrt{h}$ and
b. The mechanical power (P_{mech}) is related to the head and flow by the product $P_{mech} = A_t hq$, where A_t is a constant for the turbine/generator Unit which depends directly on its MW rating and inversely on MVA rating.

Like many other stations, the hydraulic system at Dinorwig is inherently multivariable, (except when only one Unit is in use), because the common supply conduit produces significant dynamic coupling between the turbines [3]. This is known to have an adverse effect on the stability margin in closed loop. Units that are on-line react, via their governors, to pressure and flow (and therefore power) perturbations caused by other Units whereas those that are off-line have their guide vanes closed and do not interact. The structure of the plant therefore varies with time, depending on the number of active Units. Two important nonlinearities are invariably included in the governors of hydroelectric plant :

a. A *saturation constraint* that limits the maximum guide vane opening to about 95% of the physical aperture to prevent it from hitting its end-stop. Note that a PI controller does not take the proximity of this constraint into account when an Unit is operating close to its upper limit of generated power;
b. A fixed *rate-limit* at which the guide vane can open or close that prevents excessive variation in tunnel pressure (for safety reasons and to minimise fatigue stresses on the wall material). This plays a vital role in alleviating the NMP response which occurs during the initial part of rapid power transients. It would be preferable for the controller to be responsible for this constraint so that higher guide vane rates are permitted when conditions allow.

B. *Control of hydroelectric plant*

Until recently, all control system design for hydroelectric plant was done on the basis of SISO linearised models, although simulation is usually done on a nonlinear model. Because of the nonlinearities noted in section 1.1, the parameters of a linearised model vary with the flow and head. A fixed parameter PI controller can therefore only be optimum at the operating point selected during design. A number of workers have proposed methods for improving performance across the operating envelope by adapting the controller according to the operating condition. For instance, Orelind *at al* [4] demonstrated the use of a gain-scheduled controller which selects the parameters of a PID compensator as a function of the guide vane angle. Ye *at al* [5] also describe a controller whose parameters vary over the plant's operating envelope as a function of the static head, guide vane angle and turbine speed. They also note that the dynamics are affected by the guide vane rate limit and propose a nonlinear gain term to compensate for its effect. Finally, they propose that the structure of the controller should change in order to accommodate various operational modes, e.g. frequency-control or speed-regulation.

In order to retain the convenience of SISO analysis, hydraulic coupling is sometimes modeled as an effective increase of the 'water starting constant', T_W, (which determines that rate at which the water column accelerates in the tunnel) as the number of Units on-line increases [6]. However, designing on the basis of a SISO model, with every governor tuned for the worst-case interaction (all Units on-line), leads to conservative tuning [7]. This cautious approach is understandable because exceeding the stability boundary can cause highly undesirable frequency oscillation to occur [1]. It is common, however, for hydroelectric plant to spend considerable periods with only one or two Units active, when conservative tuning leads to sub-optimal performance. This is less tolerable in an economic climate of privatised utilities.

Countering this form of plant variation has again led to consideration of adaptive controllers. Mansoor *et al* [8] have shown that open loop gain scheduling according to the number of Units on-line is a simple but reasonably effective measure. Lansberry & Wozniak [9] suggest using a genetic algorithm to perform the adaptive function so that the gains of a PI governor are made to continuously track changes in either the water starting time or the Grid stiffness. In a recent paper, Eker & Tumay [10] use the method of H_∞ optimisation to design a robust controller which is insensitive to uncertainties in plant parameters, including the water starting time and the wave travel time for an inelastic water column. Simulation results indicate that the H_∞ controller, although linear and fixed, gives better rejection of both electrical load disturbances and cross-coupled 'water' disturbances than does a PID controller. Whilst SISO design is convenient, it is clearly a compromise and it is preferable to address the effect of coupling directly by multivariable methods. Jones [11] has used a 2-input, 2-output model and the Direct Nyquist Array (DNA) technique to demonstrate the loss of stability margin caused by hydraulic cross-coupling and how a decoupling controller can be designed in order to counter this effect.

To summarise, a control method is required that takes account of multivariable effects, handles constraints and provides an integrated approach to station control according to the number of Units on-line. Model Predictive Control (MPC) [12], [13] seems to offer a solution for the following reasons:

a. It is widely acknowledged as being straightforward to extend to multivariable systems.
b. Perhaps its greatest advantage (which has stimulated its widespread acceptance in industrial process control) is that it deals naturally with hard constraints on the control and state variables, whereas linear methods (like DNA) do not.
c. It offers an integrated treatment [14] that connects control at governor level to the supervisory layer (although these aspects are not considered here).

II. MODELING, SPECIFICATION AND REFERENCE RESPONSE

A. *Plant model*

The plant model is shown in Figure 2.

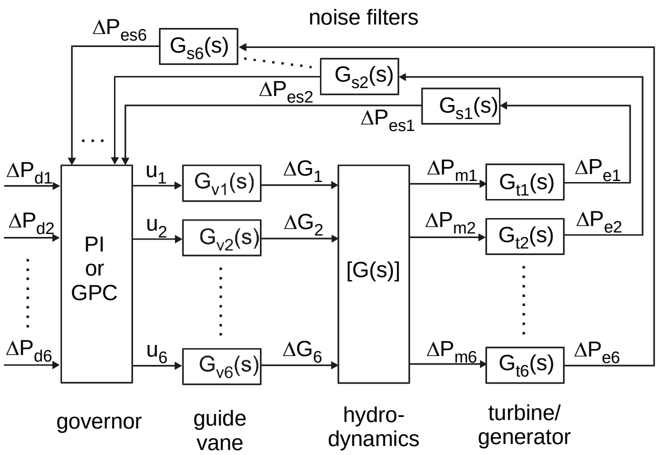


Figure 2 MIMO model of the hydroelectric plant.

Linearising the well-known multivariable model published by an IEEE Working Group [3] yields a transfer function matrix G(s), relating changes in mechanical power produced by the turbine (ΔP_m) to changes in the guide vane opening (ΔG). An inelastic water column and negligible losses are assumed and all 6 Units are taken to be identical (although minor differences due to manufacturing tolerances do occur in practice). The model is expressed in the per-unit system, normalised to 300MW and 50Hz, and assumes a Grid system with infinite busbars. The general form of the transfer function matrix [G(s)] is:

$$Y(s)=\begin{bmatrix} G_d(s) & X_c(s) & \cdots & X_c(s) \\ X_c(s) & G_d(s) & \cdots & X_c(s) \\ \vdots & \vdots & \ddots & \vdots \\ X_c(s) & X_c(s) & \cdots & G_d(s) \end{bmatrix} U(s) \qquad (1)$$

where $G_d(s)$ and $X_c(s)$ are, respectively, the direct and cross-coupling transfer functions. G(s) varies from (1 x 1) to (6 x 6) according to how many Units are on-line (M) as shown in Table 1. The rules to determine whether an unit is on- or off-line are:

Unit n comes on-line when $\Delta P_{dn} > 0$

Unit n goes off-line when $[(\Delta P_{dn} = 0)$ AND $(\Delta P_{mn}) = 0]$

In Figure 2, the guide vane opening ΔG is actuated by an oil-hydraulic servo whose transfer function is :

$$\frac{\Delta G}{u} = \frac{1}{(0.19s+1)(0.4s+1)} \qquad (2)$$

where u is the control produced by the governor. The turbine's mechanical power output ΔP_m drives the electrical subsystem whose transfer function (3) represents the well-known 'swing' equations [2] :

$$\frac{\Delta P_e}{\Delta P_m} = \frac{K_s\omega_0 / 2H}{s^2 + \frac{K_D}{2H}s + \frac{K_s\omega_0}{2H}} \qquad (3)$$

where H is the turbine/generator inertia constant, K_s is the synchronising torque coefficient, K_D is the damping coefficient and ω_0 is the base rotor electrical speed. A first order filter for noise reduction is included in each power feedback loop, which has the transfer function:

$$\frac{\Delta P_{es}}{\Delta P_e} = \frac{1}{s+1} \qquad (4)$$

The current governor comprises an individual PI controller on each turbine, of the form:

$$u = \left(K_p + \frac{K_i}{s}\right)(\Delta P_d - \Delta P_{es}) \qquad (5)$$

This model was validated across the operational range by comparison of small-step responses with the full nonlinear simulation of Dinorwig, which has itself been validated against measured responses [1].

B. *Response specification for hydroelectric plant*

The role of a hydroelectric station in frequency control mode is to provide accurate and timely supply of its target power contribution to the Grid. The actual power demanded depends on frequency error and droop but, for testing, it can be specified in terms of step, ramp and random input signals [15]. In this paper, the step response specification for single unit operation, expressed in Figure 3 and Table 2, will be used (these are not valid for commercial purposes).

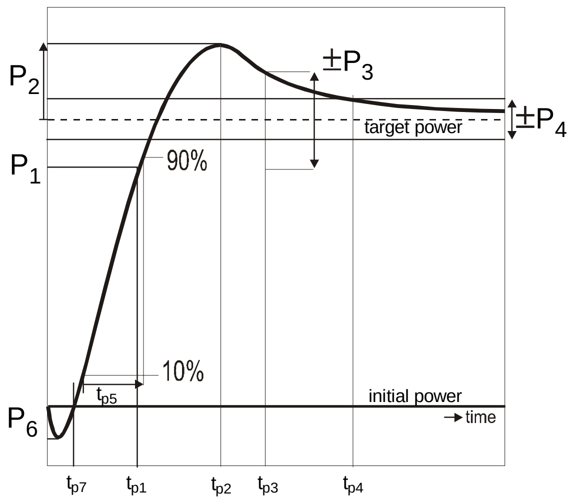


Figure 3 Specifications for a response to a step change in demanded power.

TABLE 1 VARIATION OF TRANSFER FUNCTION MATRIX WITH NUMBER OF ACTIVE UNITS (0.95 OPERATING POINT)

M	$G_d(s)$	$X_c(s)$
1	$\frac{-2.358s+3.395}{0.076s^3+0.8204s^2+2.788s+3.031}$	0
2	$\frac{-2.358s^2-5.454s+14.96}{0.076s^4+1.26s^3+7.213s^2+16.69s+13.35}$	$\frac{-8.559s}{0.076s^4+1.26s^3+7.213s^2+16.69s+13.35}$
3	$\frac{-2.358s^2-1.986s+11.01}{0.076s^4+1.221s^3+6.643s^2+14.1s+9.83}$	$\frac{-6.301s}{0.076s^4+1.221s^3+6.643s^2+14.1s+9.83}$
4	$\frac{-2.358s^2+0.03428s+8.711}{0.076s^4+1.198s^3+6.311s^2+12.59s+7.778}$	$\frac{-4.985s}{0.076s^4+1.198s^3+6.311s^2+12.59s+7.778}$
5	$\frac{-2.358s^2+1.357s+7.207}{0.076s^4+1.183s^3+6.093s^2+11.6s+6.435}$	$\frac{-4.124s}{0.076s^4+1.183s^3+6.093s^2+11.6s+6.435}$
6	$\frac{-2.358s^2+2.289s+6.145}{0.076s^4+1.173s^3+5.94s^2+10.9s+5.487}$	$\frac{-3.517s}{0.076s^4+1.173s^3+5.94s^2+10.9s+5.487}$

TABLE 2 SPECIFICATION FOR ADVANCED CONTROL DESIGN AT DINORWIG.

Test	Specification for single unit operation.	Single unit response with current governor.
P1	$P_1 \geq 90\%$ at $t_{p1} = 10s$	81% at 10s, 90% at 13.3s
P2	$P_2 \leq 5\%$ and $t_{p2} \leq 20s$	No overshoot
P3	$t_{p3} = 25s$ for $P_3 \leq 1\%$	25s
P4	$t_{p4} = 60s$ for $P_4 \leq 0.5\%$	29s
P5	$t_{p5} = 8s$	11.8s
P6	$P_6 = 2\%$	6.3%
P7	$t_{p7} = 1.5s$	1.2s

The most important criterion is usually the *primary response* (P1), which requires that the station, under defined conditions, achieve at least 90% of the demanded step power change within 10s of initiation. Table 2 also shows that the over-shoot P_2 must not exceed 5% and the initial negative excursion P_6 (the NMP response) must not exceed 2%.

C. Response with the current governor

The response of the model of section IIA, with the PI controller at its standard settings of $K_i = 0.12$ and $K_p = 0.1$, is summarized in Table 2 and the dotted graphs in Figure 5. Note here that this is the unconstrained case where the guide vane rate limit has been omitted temporarily – it will be introduced in section IV. As shown in Figure 5, when only one Unit is operational (all other Units being off-line), the response is rather slow and very well-damped. Comparison with the specification in Table 2 shows that several criteria are not satisfied. Also shown in Figure 5 is the case of 6 Units operating together, initially at 86% of full load. Simultaneous application of a 0.04p.u. step demand results in a large overshoot because the hydraulic coupling has increased the effective water starting time, T_W. It is clear that the standard PI controller setting is a compromise between the two extremes. Increasing the loop gain would improve the 1-Unit response but make the 6-Unit response even worse. The aim of a GPC controller should therefore be to satisfy the single-unit specification and also achieve fast, well-damped and low-interaction responses during multi-unit operation.

III. MIMO GENERALISED PREDICTIVE CONTROL

A. Unconstrained GPC

Although they all adhere to a common principle, there are many variants of MPC. One of the best-known is generalized predictive control (GPC) [16], which was used in this work and is outlined briefly here. The fundamental idea is to include within the controller a relatively simple 'predictive model' of the plant. Based on present and past values of the control and measured outputs of the plant, this model is used to predict the future output of the plant if subjected to a given control input. This is done for a specified number of samples (N_u) into the future, known as the 'control horizon'. The predicted error of the output from some reference trajectory can then be computed. An optimization problem is set up and solved for the future control which minimizes a cost functional of the plant error and the control exerted (these being conflicting requirements). Only the first of the optimized control values is actually applied to the physical plant because, at the next sample, the complete procedure is performed once more to generate a new control. The quadratic cost function to be minimized at each sample time *t* is given by:

$$J = \sum_{j=N_1}^{N_2} \left[\hat{y}(t+j\,|\,t) - w(t+j)\right]^2 \bar{Q} + \sum_{j=1}^{N_u} \left[\Delta u(t+j-1)\right]^2 \bar{R} \quad (6)$$

where ŷ(t+j|t) is the optimum predicted output of the plant at sample *j*, Δ is the $(1\text{-}q^{-1})$ operator, N_1 and N_2 are the start and end of the output prediction horizon, w(t+j) is the future reference trajectory and R and Q are positive definite weighting matrices which trade-off output accuracy and control effort.

The discrete-time predictive model is conveniently represented in the Model Controller Auto-Regressive Moving-Average with Integrator (CARIMA) form with *m* inputs, *n* outputs and input delay *d*:

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t-1) + C(q^{-1})\frac{e(t)}{1-q^{-1}} \quad (7)$$

In (7), $\mathbf{A}(q^{-1})$ and $\mathbf{C}(q^{-1})$ are matrices of monic polynomials of order n x n and $\mathbf{B}(q^{-1})$ is a polynomial matrix of order n x m.

When there are no constraints on the plant states or control, GPC provides a computationally efficient solution for the optimum control signal:

$$u^* = \arg\min_u (J) \quad (8)$$

It may be shown [12] that the best prediction of the output at sample *j*, given information up to and including sample *t*, is:

$$\hat{y}(t+j|t) = G_j(q^{-1})\Delta u(t+j-d-1) + F_j(q^{-1})y(t) \quad (9)$$

where G_j is a lower triangular matrix of dimension N x N (where $N = N_2 - N_1$) whose elements are points on the step response of the plant. In (9), *F* is the free response of the plant, which is calculated recursively using:

$$f_{j+1} = q\left(I - \tilde{A}(q^{-1})\right)f_j + B(q^{-1})\Delta u(t+j) \quad (10)$$

with $f_0 = y(t)$, $\tilde{A}(q^{-1}) = \Delta A(q^{-1})$ and $\Delta u(t+j) = 0$ for $j \geq 0$

In GPC, the control value is fixed for j > N_u and the form of the model predictions is then:

$$y_{N_{12}} = \left[\hat{y}(t+N_1|t)^T, \hat{y}(t+N_1+1|t)^T, \cdots, \hat{y}(t+N_2|t)^T\right] \quad (11)$$

The output predictions (9) can be expressed as:

$$y_{N12} = G_{N123}u_{N_u} + f_{N12} \quad (12)$$

where $u_{N_u} = \left[\Delta u(t)^T \ldots\ldots \Delta u(t+N_u-1)^T\right]^T$

$$f_{N12} = \left[f_{N_1}{}^T, f_{N_1+1}{}^T, \cdots, f_{N_2}{}^T \right]^T$$

and G_{N123} is a sub-matrix of G, with $G_i = 0$ for $i<0$:

$$G_{N123} = \begin{bmatrix} G_{N_1-1} & G_{N_1-2} & \cdots & G_{N_1-N_u} \\ G_{N_1} & G_{N_1-1} & \cdots & G_{N_1+1-N_u} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N_2-1} & G_{N2-2} & \cdots & G_{N_1-N_u} \end{bmatrix} \tag{13}$$

Then the cost function (6) can be rewritten as:

$$J = \left(G_{N123}u_{N_u} + f_{N12} - w\right)^T \overline{R}\left(G_{N123}u_{N_u} + f_{N12} - w\right) + u_{N_u}^T \overline{Q} u_{N_u} \tag{14}$$

where $\overline{R} = RI$ and $\overline{Q} = \lambda I$. Setting R = 1, u* can then be calculated for any fixed value of the tuning parameter λ as:

$$u^* = \left(G_{N_{123}}^T \overline{R} G_{N_{123}} + \overline{Q}\right)^{-1} G_{N_{123}}^T \overline{R}\left(w - f_{N_{12}}\right) \tag{15}$$

B. MPC in electric power generation.

Several examples of MPC applied to conventional power plant have appeared over the last few years. Recently, Prasad *at al* [17] describe controlling a thermal (boiler-turbine) power plant using a hierarchical MPC approach. Here, two of the plant's loops are controlled by local PIDs whereas the relatively slow interaction and optimisation of the overall plant is dealt with by MPC. Excellent disturbance-rejection and alleviation of plant-wide interactions are reported for simulations performed on a 200MW power plant. Rossiter *et al* [18] also consider MPC for a fossil-fired power station where the design is constrained by limitations on the hardware available for implementation. They proceed to derive a slightly sub-optimal algorithm called efficient constrained GPC (ECGPC) with the interesting conclusion that the output of the plant is marginally improved, compared to using a PID, but control activity is substantially reduced.

To date, there has been little interest in applying MPC to power and frequency control of hydroelectric plant (which have very different characteristics and time-scales to thermal generation plant). Sansevero and Bottura [19] considered a small-perturbation SISO model relating turbine speed to input power. Their results indicate that, provided a prediction horizon of adequate length (compared with T_W) is chosen, MPC allows less conservative tuning than PI, while retaining closed loop stability. They concluded that even linear MPC is competitive with the standard PI controllers installed at two stations currently in operation. Recently, Muñoz Hernández and Jones described the application of GPC to a linearised SISO model of the Dinorwig station [20]. The results indicated that GPC gives smoother, faster and better-damped responses over the operating range of the plant and encouraged the multivariable development described here.

C. The predictive model

Two types of predictive model (7) were investigated. The first was an analytical model consisting of only the hydraulic and guide-vane subsystems (G(s) and G_v(s)). For instance, in the case of 6 units operating at G_o=0.95 and a sample period T_s = 0.25s the direct and cross-coupling transfer functions are:

$$G_6(z) = \frac{-0.2312z^3 + 0.5145z^2 - 0.1481z - 0.07865}{z^4 - 1.802z^3 + 1.093z^2 - 0.2616z + 0.02113} \tag{16}$$

$$X_6(z) = \frac{-0.04757z^3 - 0.02751z^2 + 0.06817z + 0.006913}{z^4 - 1.802z^3 + 1.093z^2 - 0.2616z + 0.02113} \tag{17}$$

The corresponding polynomials in (7) are:

$$A_6(z^{-1}) = 1 - 3.6042z^{-1} + 5.4339z^{-2} - 4.4631z^{-3} + 2.1801z^{-4} - 0.6481z^{-5} + 0.1146z^{-6} - 0.0111z^{-7} + 0.0004z^{-8} \tag{18}$$

$$B_6(z^{-1}) = -0.2312z^{-1} + 0.9312z^{-2} - 1.328z^{-3} + 0.8111z^{-4} - 0.1596z^{-5} - 0.0364z^{-6} + 0.0174z^{-7} - 0.0017z^{-8} \tag{19}$$

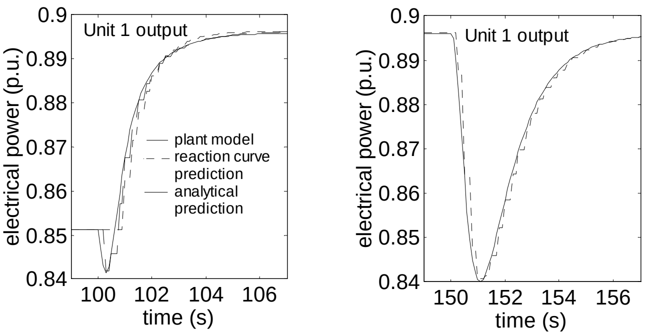
$$b_6(z^{-1}) = -0.0476z^{-1} + 0.0582z^{-2} + 0.0658z^{-3} - 0.1336z^{-4} + 0.0682z^{-5} - 0.0109z^{-6} - 0.0004z^{-7} + 0.0001z^{-8} \tag{20}$$

A_6, B_6 being the diagonal and b_6 the off-diagonal elements.

In the second predictive model, (16) was replaced by a simpler transfer function calculated from the reaction curve:

$$G_{6c}(z) = \frac{0.29044}{z^4 - 0.7422} \tag{21}$$

In fact, the predictions produced by both models are similar and agree well with the plant output shown in Figure 4. GPC controllers were designed using both (16) and (21) as the direct transfer function in the predictive model. The results were similar, so the more economical transfer function (21) was used for the remainder of the work.



(a) (b)

Figure 4 Response of the Plant model and the predictive models to a 0.04 p.u. step in control (U), (a) applied to Unit 1, with the controls for Units 2-6 fixed at their initial values and (b) applied simultaneously to Units 2-6, with the control for Unit 1 fixed at its initial value.

D. Unconstrained GPC results

The response produced by a GPC controller with N_u = 20, N = 40, α = 0 and λ = 425 is compared with that of the PI controller in Figure 5, for the cases described in section IIC. For the 1-Unit case, Table 3 shows that GPC produces a primary response that is 12% faster, settles 14% sooner and has only 30% of the NMP undershoot produced by PI control. GPC also produces a smoother control that inhibits the rapid, poorly-damped synchronous electrical mode which the relatively sharp PI control excites.

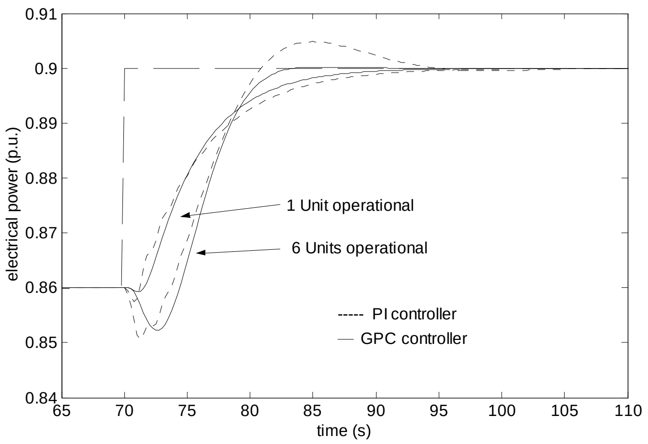


Figure 5 Comparison of the step responses produced by the GPC design and the standard PI controller for the 1-Unit and 6-Unit operational cases.

In the 6-Unit case, GPC almost eliminates the overshoot produced by the PI controller and also reduces the NMP undershoot, with no adverse effect on primary response. It is worth stressing that neither controller (or any other controller of practical interest) can eliminate the NMP undershoot. As the guide vanes open, part of the mechanical power is diverted to accelerating the water column and is denied to the output - a fundamental physical limitation. The controller may therefore distribute the power shortfall in time (by opening the guide vane slower) or by 'borrowing' from other turbines (decoupling control [11]) but has no way to make up the transient power deficiency.

TABLE 3 COMPARISON OF PI AND GPC SINGLE-UNIT RESPONSES

Test	PI	GPC (unconstrained)	GPC (constrained)
P1	81% at 10s, 90% at 13.3s	85% at 10s, 90% at 11.7s	92% at 10s, 90% at 9.3s
P2	No overshoot	No overshoot	No overshoot
P3	25s	21.4s	16.5s
P4	29s	24.3s	19s
P5	11.8s	9.4s	7.1s
P6	6.3%	2%	2.5%
P7	1.2s	1.54s	1.54s

Figure 6 shows the cross-coupling responses of the two control systems. Initially, all 6 Units are operating at 0.86p.u. At t = 100, a 0.04p.u. step demand is applied to Unit 1. The consequent perturbation on the outputs of the remaining 5 Units is reduced by using GPC. At t = 140, a simultaneous step demand is applied to Units 2 - 6. Here, GPC eliminates the overshoot, reduces the initial under-shoot and offers a small improvement in the cross-coupling onto Unit 1. Whilst the PI controller can be re-tuned to reduce the overshoot, it was found that this always slowed the response and caused a larger initial under-shoot. It is concluded that unconstrained GPC is superior to PI across the operating range of the plant.

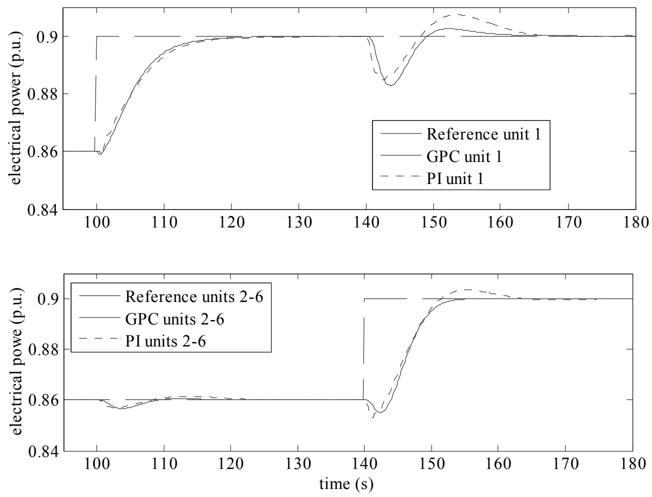


Figure 6 Responses of unconstrained GPC and PI to 0.04 p.u. step power demand for Unit 1 (upper) and Units 2 – 6 (lower).

IV. CONSTRAINED GPC.

Although unconstrained GPC is superior to PI, the performance indicators in Table 3 fall short of the specification in Table 1. Reducing the value of λ yields a faster response but extensive simulation showed that it is always at the expense of other criteria. In this section, constraints on the guide vane rate and amplitude are incorporated within the GPC framework. This allows the 1-Unit specification to be satisfied while preserving stability during multiple unit operation.

A. Constrained MPC theory

In predictive control three kinds of constraints are normally considered - saturation limits on the controls and outputs and rate limits on the controls [12], [13]. These can be represented as follows:

Control limit: $\underline{u} \le u(t) \le \overline{u} \quad \forall t$

Control rate limit: $\underline{\Delta u} \le u(t) - u(t-1) \le \overline{\Delta u} \quad \forall t$

Output limit: $\underline{y} \le y(t) \le \overline{y} \quad \forall t$

Expressing (6) in the form:

$$J(u) = \frac{1}{2}u^T H u + b^T u + f_0 \tag{22}$$

where $f_0 = (f_{N12} - w)^T (f_{N12} - w)$, $b^T = 2(f_{N12} - w)^T G_{N123}$

and $H = 2(G_{N123}^T G_{N123} + \lambda I)$,

allows the predictive controller to be posed as a Quadratic Programming problem, where:

$$J(u) = \frac{1}{2}u^T H u + b^T u \tag{23}$$

is to be optimized, subject to the control constraints $A_{qp} u \le b_c$. The term f_0 is excluded from (23) because, at every stage of optimization, it is a constant whose value is independent of *u*, and every value of *J(u)* and *u* are non-negative. The ($6N_u$ x m) matrix A_{qp} and the vector b_c of length $6N_u$ are defined:

$$A_{qp} = \begin{bmatrix} I \\ -I \\ I \\ -I \\ G_{N_{123}} \\ -G_{N_{123}} \end{bmatrix} \quad b_c = \begin{bmatrix} \underline{1}(\bar{u}-u) \\ \underline{1}(\underline{u}+u) \\ \underline{1}\overline{\Delta u} \\ \underline{1}\underline{\Delta u} \\ \underline{1}\bar{y}-f \\ -\underline{1}\underline{y}+f \end{bmatrix} \quad where \quad \underline{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad (24)$$

Optimization of (23) is done at each sample instant using an interior-reflective Newton method and the unconstrained solution as an initial guess. This is an iterative method and increases the computation time substantially.

B. Constrained GPC (CGPC) results

The control rate-limit is fixed at $-0.2 \le \Delta u \le 0.2$p.u., approximately twice its current value, allowing the guide vanes to move more rapidly when necessary. An effective strategy for the control saturation limit is to cap its value at each sample instant according to:

$$\begin{aligned} 0 \le u \le P_d / A_t \qquad & \dot{P}_d > 0 \\ P_d / A_t \le u \le 1/A_t \qquad & \dot{P}_d < 0 \end{aligned} \qquad (25)$$

i.e. the control is never allowed to exceed the steady-state value required by the demanded power. Re-tuning the CGPC controller yielded the new parameters $N_u = 10$, $N = 40$, $\alpha = 0$ and $\lambda = 350$ (compared to $\lambda = 425$ for the unconstrained case, thus effectively increasing the 'loop gain').

To make a fair comparison, the PI controller was also modified to include control constraints. The same rate limit of 0.2p.u. was used and the saturation limit fixed to u = 1p.u. in the anti-windup configuration shown in Figure 7.

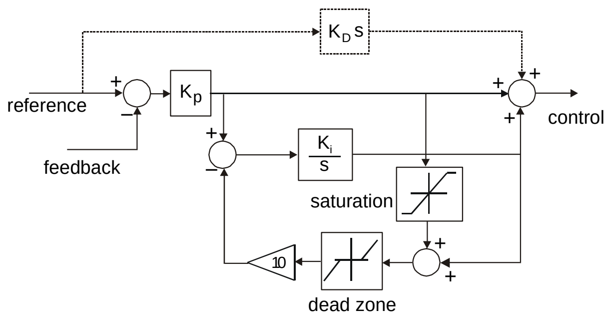


Figure 7 Anti-windup configuration for the modified PI controller, also showing the derivative feed-forward path.

The PI parameter values were chosen as K = 0.165 and K_i = 0.65, to give the best response for the 6-Unit case. The results are shown in Figure 8, which correspond to the cases in Figure 5. The CGPC response for the 1-Unit case is faster than the unconstrained case but remains well damped and has a very small NMP under-shoot. Its performance indicators (last column of Table 3) comply with the specification in Table 1 except for test P6. In contrast, the response produced by the modified PI controller is barely faster than the unconstrained case and the electrical oscillation is prominent. In fact, this prevents any further increase in the PI loop gain, although it has been found that addition of a derivative term into the power loop alleviates the effect. The advantage of the CGPC controller is retained in the 6-Unit case, producing a smooth, fast response with no overshoot despite the change in operating conditions. Awareness of the constraints within the predictive model allows CGPC to make informed decisions about their effect and produce better controls.

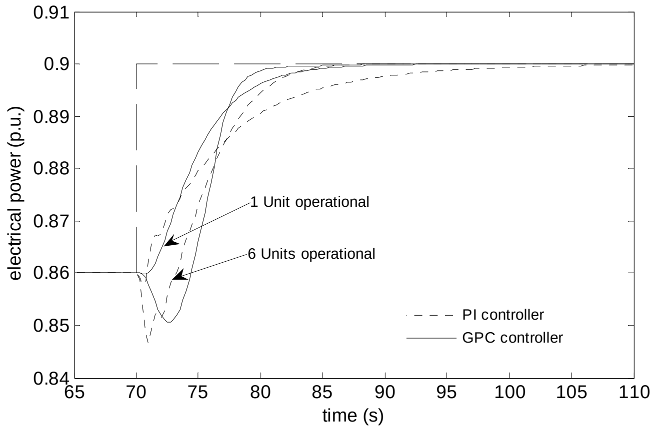


Figure 8 Comparison of the step responses produced by the CGPC design and the modified PI controller for the 1-Unit and 6-Unit operational cases.

Hydro-governors commonly include a derivative term in the feed-forward path from the reference (see dotted path in Figure 7). This produces a rapid change in power in response to rapidly-changing frequency. Using the standard Dinorwig derivative gain of K_D = 0.2, Figure 9 shows that the PI controller's response advances by approximately 0.5s during the early part of a ramp reference input. The same strategy can be combined with CGPC too, with similar effect. Once the ramp ends, the advantage of CGPC is re-asserted.

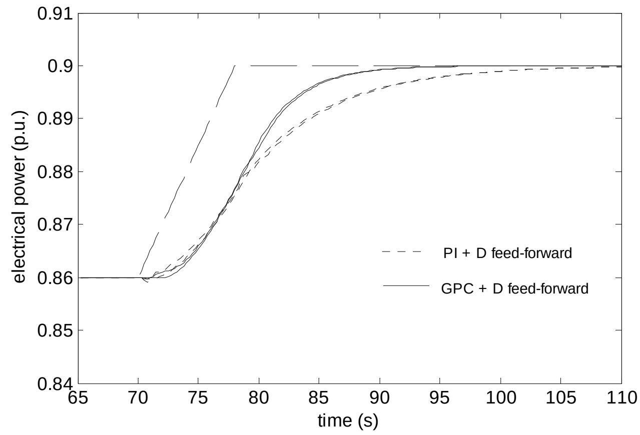


Figure 9 Comparison of the responses for the PI and CGPC designs when the derivative feed-forward loop is included (1-Unit operational case).

The cross-coupling responses for CGPC and the modified PI are shown in Figure 10, where it is seen that the over-shoot is improved in both direct and coupled transients, with little effect on the speed of response. However, as discussed previously, the NMP under-shoot remains.

V. Conclusions

The basic question of whether GPC can improve the control of fast-response hydroelectric power stations has been answered positively in this paper. Taking explicit account of the multivariable nature of the plant improves both the direct and cross-coupled transient responses compared with PI control. Inclusion of a rate constraint in the GPC controller yields a fast, well-damped response in the common case when only a single Unit is in operation, without compromising stability when multiple Units are on-line. Simulation has also shown that improved power delivery is obtained when the plant is operated in frequency control mode.

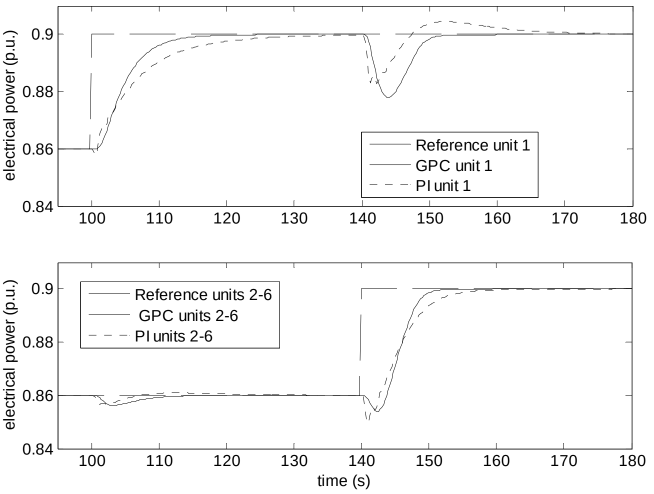


Figure 10 Responses of constrained GPC and PI to 0.04 p.u. step power demand for Unit 1 (upper) and Units 2 - 6 (lower)

In the next stage of the work, the performance of the proposed CGPC controller will be investigated more thoroughly. It will be confirmed that it retains its performance advantage on the full nonlinear simulation of Dinorwig and its sensitivity to plant variation (e.g. lake head, Grid size) will be investigated. Current work is also investigating how CGPC can be integrated with higher level plant functions by using a mixed logical/dynamic (MLD) controller.

As for practical implementation, it is likely that any radical change in the control software will be co-ordinated with the next major revision of governor hardware, possibly some years hence. Nevertheless, this work has provided Dinorwig (and similar hydroelectric stations) with an assessment of the potential of GPC and valuable tuning guidelines.

VI. Acknowledgment

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VIII. Biographies

German Ardul Munoz-Hernandez (M'2002) received the BSc degree in Electronic Engineering from Veracruz Institute of Technology in 1988. From 1986-1987 he was a programmer in FAPATUX, a Pulp and Paper factory in Tuxtepec, Mexico. In 1990 he received the MSc degree in Electronic Systems from the National Institute of Astrophysics Optic and Electronic (INAOE, Mexico). From 1990-1993, he returned to work in FAPATUX in the area of Engineering and new projects. From 1993-1996, he was in the Electronics department at the University of Puebla, which included periods as a visiting Engineer at the University of South Carolina and the University of Michigan. He is currently at the Puebla Institute of Technology where his main areas of interest are control systems applications and automation. He received the PhD degree of the University of Wales in 2005.

Dewi Jones (M'93, SM'97) received the BSc degree in Electronic Engineering from the University of Wales, Bangor in 1972. For three years he was an Ionosphericist with the British Antarctic Survey after which he returned to the University of Wales, Bangor to take the MSc and PhD degrees in control systems engineering. He then joined the GEC Marconi Research Laboratory but returned to Bangor in 1980 as a Lecturer, then subsequently a Senior Lecturer and Reader. From 1994 -1996 he was a Royal Society Industry Fellow at EA Technology Ltd. His main area of interest is control systems applications which include control of electrical an electromechanical devices, real-time parallel computation of control algorithms, vibration isolation for spacecraft, automation of power line inspection and hydroelectric plant control. In 2002, he was awarded the DSc degree by the University of Wales.